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A formal modeling approach for supply chain event management

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Abstract

As supply chains become more dynamic, there is a need for a sense-and-respond capability to react to events in a real-time manner. In this paper, we propose Petri nets extended with time and color (to represent case data) as a formalism for managing events. We designed seven basic patterns to capture modeling concepts that arise commonly in supply chains. These basic patterns may be used by themselves and also combined to create new patterns. We also show how to combine the patterns to build a complete Petri net and analyze it using dependency graphs and simulation. Dependency graphs can be used to analyze the various events and their causes. Simulation was, in addition, used to analyze various performance indicators (e.g., fill rates, replenishment times, and lead times) under different strategies. We showed it is possible to perform sensitivity analysis to study the effect of changing parameter values on the performance indicators. This approach thus makes a very complex problem tractable.

Keywords: Supply chain event management (SCEM); Petri nets; Time Petri nets; Colored Petri nets; Event causality; Dependency graph; Reachability; CPN tools

1. Introduction

The pressures of global competition and the need for extensive inter-organizational collaboration are forcing companies to streamline their supply chains and make them agile, flexible and responsive. Consequently, a supply chain must be able to handle large numbers of events, both expected and unexpected. The unexpected events, also called *exceptions*, typically arise because there is usually a gap between supply chain planning and execution [2]. Supply chain planning sets a target that can be achieved based on a given set of constraints at a given time. In a dynamic supply chain environment, the constraints are always changing, so exceptions or deviations from plans occur almost regularly. Examples of exceptions are product out-of-stock, shipment delay, machine breakdown, etc., and they are costly. Moreover, events tend to propagate in collaborative supply chains across partners, resulting in the well-known bullwhip effect [13]. Such risks have given rise to a new research area of Supply Chain Event Management (SCEM). The goal of SCEM is to introduce a control mechanism for managing events, in particular, exception events, and responding to them dynamically.

A supply chain event is "any individual outcome (or non-outcome) of a supply chain cycle, (sub) process, activity, or task" [1]. Events are correlated with each other to form a "cloud" of events; some events have

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significant consequences and, therefore, they must be monitored closely, while others are of lesser importance. The critical problem lies in extracting the significant events and responding to them in real-time. Doing so requires an ability to monitor them proactively, to simulate them to help decision-making and to use them to control and measure business processes [21,26]. In this paper, we present a methodology that uses a Petri net approach to formulate supply chain event rules and to analyze the cause–effect relationships between events.

Petri nets are a powerful modeling technique for problems involving coordination in a variety of domains. A variant of Petri nets called *time Petri nets* allows us to model time intervals also. Considering the dynamic characteristic of supply chain events, such Petri nets are useful for describing the time constraints associated with events. Examples of time constraints are: "*event* e_1 *follows event* e_2 *after time T*" and "*N occurrences of event* e_1 *within time T lead to event* e_2." These temporal constraints are important for proper correlation between events; otherwise, the management would be unable to anticipate events or track causes of events. To deal with variety in case data (e.g., order ids, order quantities, rush orders versus normal orders, etc.) we extend the model with "token colors", i.e., we use *time colored Petri nets*.

Using time colored Petri nets we can model event patterns common in Supply Chain Management (SCM). These patterns can be composed as demonstrated using a Vendor Managed Inventory (VMI) example. To demonstrate that the Petri net basis allows for different types of analysis we used CPN Tools [23] to simulate different scenarios for the VMI example. The mapping onto CPN Tools allows us to investigate the performance of event resolution strategies. In addition, dependency graphs are used to analyze cause-and-effect relationships of events.

There is a variety of SCEM systems offered by companies, such as SAP, i2, and Manugistics [26]. Most systems mainly perform monitoring and provide "early warning" rather than analyzing events and suggesting solutions [2,4,17,19,21,26]. Actually, the more powerful part of SCEM would be the capability of "aggregating data from key business systems at a high level and presenting the ramifications of exceptions and the possibilities of solutions [17]." Therefore, this research can contribute to the research area of SCEM in three ways. First, it introduces a formal and general approach to modeling events and event rules, and the approach provides flexibility in associating occurrence counts and temporal constraints with events, avoiding the customization problem which is often an obstacle to the implementation of the existing SCEM systems [4]. Second, this approach allows excellent event analysis, including event forecasting with time information and causality analysis, which provide real-time visibility about the implications of events and traceability to the root causes for events. Third, it offers a way to track supply chain performance metrics by events, and shows how through simulation decision makers can compare different strategy alternatives or fine-tune a solution in terms of key performance indicators.

The paper is structured as follows. Section 2 gives an overview of events, event rules, event aggregation, event causality, and our notion of a dynamic supply chain. Section 3 describes Petri nets briefly. Section 4 introduces event semantics and seven event patterns or building blocks of event rules. It shows that a complete event Petri net can be constructed easily by using these blocks. Section 5 presents a detailed example to illustrate how to use Petri nets to examine event causality and forecast subsequent events. Section 6 gives simulation results of the example to illustrate the practical value of our approach. Section 7 describes related work and compares our approach with others, while Section 8 concludes the paper with a brief description of future work.

2. Overview of supply chain events

When supply chain partners are integrated, events at one partner may have impact on other partners, and their responses to these events may cause a storm of events. Therefore, causality analysis is the key to controlling such a storm. Our analysis begins with events and event rules.

In general, events in an organization are of three types: (1) *Task status related events,* such as the end of a task or the beginning of a task. These events are usually regular; (2) *Events produced by a task*: for example, events "stock partially available" and "out of stock" are the result of the "check availability" task; and, (3) *External events* which may arrive from other supply chain partners or from the external environment, e.g., *new order arrival, inbound shipment delay, import policy change* etc.

These types of events are captured directly during a process, and called *simple or primitive* (as opposed to *composite*) *events*. Composite events are *derived* from simple events by *event aggregation*. A composite event is deduced when a group of simple events occurs [16]. A group of simple events may together reveal potential problems. For example, if a product is out of stock once

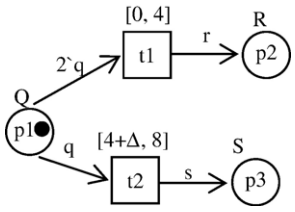


Fig. 1. Colored time Petri net.

in a month, perhaps it is quite normal and an alarm should not be generated, but *if this stock out happens two times in a week, then it may reflect some underlying problems in the product supply chain and this should be recognized by generating an event.* As another example, a group of stock trading events, related by accounts, timing and other data, taken together, may constitute a violation of a policy or a regulation [16]. *Event aggregation* is a mechanism to filter simple events and extract meaningful information from them by setting up alarms in advance.

Thus, *event aggregation* extracts value from a management point of view out of trivial and unorganized simple events. In order to achieve this objective, it is important to recognize event patterns and set up *aggregation rules*. Besides aggregation rules, *business rules* must also be considered. Business rules capture the causal relationships between events. For example, if an order is delayed for more than time *T*, then it is automatically cancelled. Therefore, a rule is needed to express that the event "order delayed by *T*" is a cause of event "order cancelled".

Moreover, a supply chain is viewed as a series of synchronous and asynchronous interactions among trading partners. Usually, when an event, particularly an exception, happens, the trading partner responsible for it may react to this event within a reasonable *resolution time* to resolve it. For instance, suppose an order is delayed for delivery. If the delay is within an acceptable range specified by the customer, the customer is notified of the delay and the order is processed. However, if the delay exceeds the acceptable tolerance (also called *expiration time*), the order should be automatically cancelled, and hence, the event "order delay" is not relevant in this case. On the other hand, a series of new actions may also arise because of this new event, such as canceling the order, removing any reservations made, refunding any payments, etc. Therefore, to model events and event rules precisely, our modeling approach should be able to capture such temporal constraints correctly. In our analysis, two time values are associated with an event: *resolution time* and *expiration time*. If an event is not resolved in a finite amount of time, it is said to expire. We will show how to capture the dynamic aspect of events in later sections.

3. Petri net preliminaries

A Petri net is a directed graph consisting of two kinds of nodes called *places* and *transitions*. In general, places are drawn as circles and transitions as boxes or bars. Directed arcs connect transitions and places either from a transition to a place or from a place to a transition. Arcs are labeled with positive integers as their weight (the default weight is 1). Places may contain tokens. In Fig. 1, one token is represented by a black dot in place $p1$. A marking is denoted by a vector M, where its pth element $M(p)$ is the number of tokens in place p. The firing rules of Petri nets are [22]:

(1) A transition t is *enabled* if each input place of t contains at least $w(p,t)$ tokens, where $w(p,t)$ is the weight of the arc from p to t. By default, $w(p,t)$ is 1.
(2) The *firing* of an enabled transition t removes $w(p,t)$ tokens from each input place p of t, and adds $w(t,p)$ tokens to each output place p of t, where $w(t,p)$ is the weight on the arc from t to p.

There is another special type of arc called the inhibitor arc with a small circle rather than arrow at the end. An inhibitor from a place to a transition prohibits the transition from being enabled, and thus firing, if there is a token in the place. An example of an inhibitor arc is given later.

In this paper, we use *Time Colored Petri Nets* (TCPN), i.e., Petri nets extended with *time intervals* and *token values*. First of all, the above classical Petri nets can be extended by associating a time interval $[I_1, I_2]$ with each transition, where I_1 (I_2) is the *minimum* (*maximum*) time the transition must wait for before firing *after* it is enabled. Such a Petri net is known as Time Petri net (TPN) [29]. If $I_1=I_2$, we just associate one time value with each transition[2], while if the interval is not specified, then $I_1=I_2=0$. Analysis techniques for TPNs are discussed in [3,29]. Second, tokens can be tagged with data values (or a color) to create a colored Petri net (CPN) [11,12]. For example, we use tokens of different colors (or values) for each order or product. For a given place, all tokens must be from one color set.

In Fig. 1, Q, R and S represent different color sets. q, r, and s are variables, such that $q \in Q$, $r \in R$, and $s \in S$. In a TCPN the arcs are also labeled with colors.

[2] The *Time* Petri Nets discussed in this paper should not be confused with *Timed* Petri Nets. A Petri net is called *Timed* Petri net [29,32] if each transition is associated with a fixed time instead of a time interval. The two types of Petri nets have very different semantics. As discussed in [3], *Time* Petri Nets are more general than *Timed* Petri Nets.

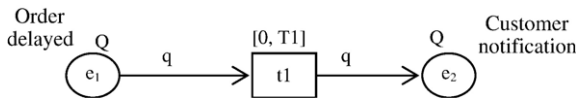


Fig. 2. Petri net of Example 1 showing a rule R.

For example, in Fig. 1, two tokens colored "q" are consumed if transition $t1$ fires. The fired transition $t1$ will put one token colored "r" in place $p2$. Moreover, if there are two tokens colored "q" continuously existing in place $p1$, transition $t1$ will fire no later than time 4. If there is still a token colored "q" remaining in place $p1$ after time 4 (relative to arrival of this token), transition $t2$ will fire shortly after time 4 (denoted as $4+\Delta$, where Δ is a very short time period, close to 0) and before or at time 8.

4. Event formulation and event patterns

4.1. Event semantics

Having given a preliminary introduction to Petri nets, we turn now to develop the techniques to formulate event related rules as Petri net structures. In most cases, events are not only the triggers but also consequences of supply chain tasks, i.e. one event causes another event. Therefore, it is quite natural to model events as places that represent pre-conditions or post-conditions of transitions. Thus, events and places will be used interchangeably while modeling events. Moreover, time Petri nets offer an attractive choice for modeling the dynamic aspect in supply chains. To make such models, we first formulate events and event rules as follows:

Event rule R: e_1 $(n_1x_1, I_0) \xrightarrow{[I_1,I_2]} e_2(x_2)$, where

- e_1 Input event class
- n_1 Number of event instances (for simplicity, we just say events), i.e., number of tokens (by default, $n_1=1$).
- x_i Data value of event i for $i=1,2$. In other words, the color of tokens, $x_i \in$ color set X_i.
- I_0 Expiration time of e_1.
- $\rightarrow$ "Imply" or "lead to", which establishes a cause–effect relationship between the left side and right side of the rule.
- $[I_1, I_2]$ An optional time interval which corresponds to the event resolution time. In order not to make the problem trivial, we require $I_2<I_0$. If this interval is not specified, we assume $I_1=I_2=0$
- e_2 Output event class. For every rule, only *one* instance of e_2 is generated because it is not necessary to repeat supply chain events.

This event rule shows the semantics of event e_1 succinctly. Suppose e_1 continues to arrive in a system. If the number of its occurrences reaches a threshold, say n_1, and these events persist in the system long enough, event rule R can be triggered during interval $[I_1, I_2]$, and e_2 is then generated. I_0 is the expiration time of e_1. If rule R does not fire within the $[I_1, I_2]$ interval, then e_1 expires. After e_2 occurs, e_1 may normally be consumed by rule R. However, if e_1 is required by another rule, then a token should be returned to e_1. Hence, two representations are possible for event rules:

Representation 1 (*consumption case* — e_1 is consumed): This case can be modeled as a Petri net shown in Fig. 2. This representation is useful when an event is not required by multiple rules.

Representation 2 (*non-consumption case* — e_1 is not consumed): Event e_1 is not consumed because it may be required by another rule. Nevertheless, event e_2 must not be generated multiple times from these occurrences of e_1. This case can be accurately modeled as a Petri net as shown in Fig. 3.

When comparing Figs. 2 and 3, one can note several differences. First of all, the representation chosen in Fig. 3 abstracts from color sets, and focuses on timing issues and causalities. Second, the events are not consumed. Third, we consider the situation where n_1 occurrences of e_1 trigger e_2. Since event e_1 is not consumed by rule R, we need a special mechanism to prevent event e_2 from being generated repeatedly. Therefore, as Fig. 3 shows, Place e_1 is first *transformed* into two places, e_1' and e_1'', through a transition $t1$. Tokens in e_1'' are consumed if transition $t2$ fires, while n_1x_1 tokens (denoted as $n_1'x_1$) are brought back to place e_1'. (Note that n_1 events are needed to enable transition $t2$.) Therefore, after the first firing, although there are n_1x_1 tokens in place e_1', transition $t2$ cannot fire, and thus, at most one e_2 event is generated (with respect to n_1x_1 tokens). If transition $t2$ does not fire (because of insufficient tokens in e_1'), e_1'' expires at the end of expiration time by the firing of transition $t3$. The notion of

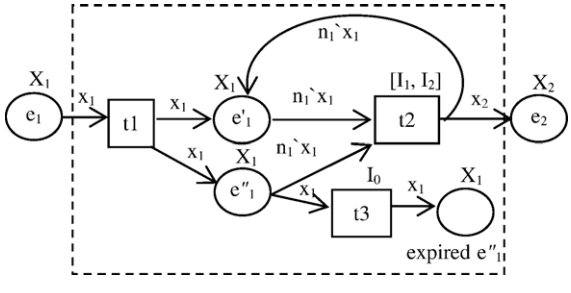


Fig. 3. Petri net representation for non-consumption case.

expiration time will be discussed further in the next section where these two representations are also employed in the patterns.

4.2. Event patterns to model supply chain rules

Next we will develop several patterns for constructing complex temporal event relationships and also give equivalent logical expressions for these patterns. In general, three logic connectives, OR ($\vee$), AND ($\wedge$), Negation ($\neg$), can be used on either the left or the right side of an event rule. Since modeling of time is crucial in understanding the behavior of our Petri net models, we call these patterns *temporal event patterns*.

We will show later that these patterns can be used as building blocks to create event networks in supply chains. We will demonstrate that these patterns allow us to capture sophisticated relationships involving multiple event instances, event expiration times and resolution times. Thus, this modeling approach can be used to model large varieties of typical supply chain events. We will also illustrate the patterns by examples.

[P1] Pattern 1 (simple cause–result pattern): A *cause–result pattern* is the most basic pattern for describing event relationships. It shows that event e_1 can cause event e_2 within a time period $[I_1, I_2]$. More formally, this relationship is expressed as: $e_1(x_1) \xrightarrow{[I_1,I_2]} e_2(x_2)$.

Example 1: If an order is delayed (e_1), contact customer (e_2) before time $T1$, i.e., $e_1(q) \xrightarrow{[0,T1]} e_2(q)$ (Note: q is order numbers).

Fig. 2 (in the previous section) shows the time Petri net model of this example. Note that *order numbers* can be considered as a color set here, i.e., each order has a different color. We use Q to denote this color set, and q is a variable for any order in Q. Transition $t1$ must fire within time $T1$ after it is enabled. Transition $t1$ corresponds to the action "notify customer".

[P2] Pattern 2 (Repeat_cause–one_effect pattern): This pattern concerns the case where multiple occurrences of one event within a certain time period cause another single event to occur. Formally, this relationship can be described as: $e_1(n_1x_1, I_0) \xrightarrow{[I_1,I_2]} e_2(x_2)$, where n_1 occurrences of event e_1 for instance x_1 cause event e_2 to occur. There are numerous situations where this pattern is useful.

Example 2: If a product with id s is out of stock (e_1) more than once within period $T2$, contact the supply chain manager (e_2). Formally, this rule can be represented as $e_1(2s, T2) \xrightarrow{[0,\Delta]} e_2(s)$.

This example introduces the notion of *expiration time* of events. If an event is not consumed (in this case, event e_1) by a rule, it may expire after a time interval. The Petri net model in Fig. 4 represents the time constraints pertaining to these events. Whenever tokens arrive at place e_1' and e_1'' (as a result of event e_1) transition $t2$ and $t3$ are enabled, but they cannot fire immediately. When there are two tokens arriving in place e_1' and e_1'', transition $t1$ fires immediately and produces the event e_2, "Notify SC Manager". After transition $t1$ fires, two tokens are returned to place e_1', because event e_1' may be used by other rules. However, tokens in place e_1'' are consumed, so transition $t1$ cannot fire repeatedly. Since transition firing does not take time, $t3$ is still continuously enabled. If a token stays in place e_1' for time $T2$ after its arrival, $t3$ fires and event e_1 expires. Thus, it is possible that event e_1' expires without $t1$ firing if there is only one token arriving within interval $T2$. Simultaneously, transition $t2$ fires so that e_1'' expires.

[P3] Pattern 3 (Inclusive choice): The need for this construct arises when multiple, alternative events can occur based on temporal conditions. Formally, this rule can be expressed as:

$$e_0(n_0x_0, I_{00})\{[\xrightarrow{[I_{11},I_{12}]} e_1(x_1)] \vee [\xrightarrow{[I_{21},I_{22}]} e_2(x_2)] \vee \ldots \vee [\xrightarrow{[I_{m1},I_{m2}]} e_m(x_m)]\}$$

Thus, in general, an event e_0 could produce one of many events ranging from e_1 to e_m based on the time intervals associated with these events. In general, these

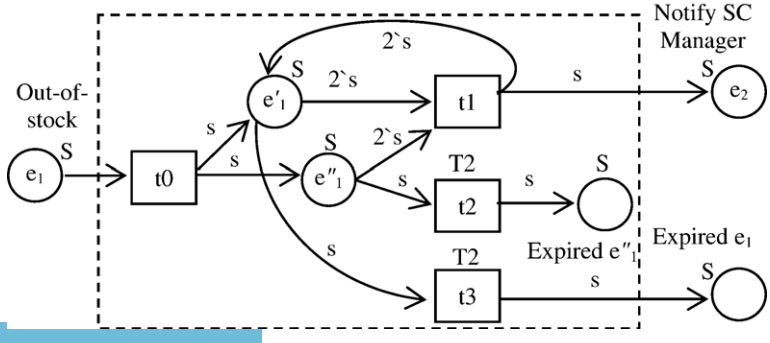


Fig. 4. Petri net of Example 2 (Pattern 2).

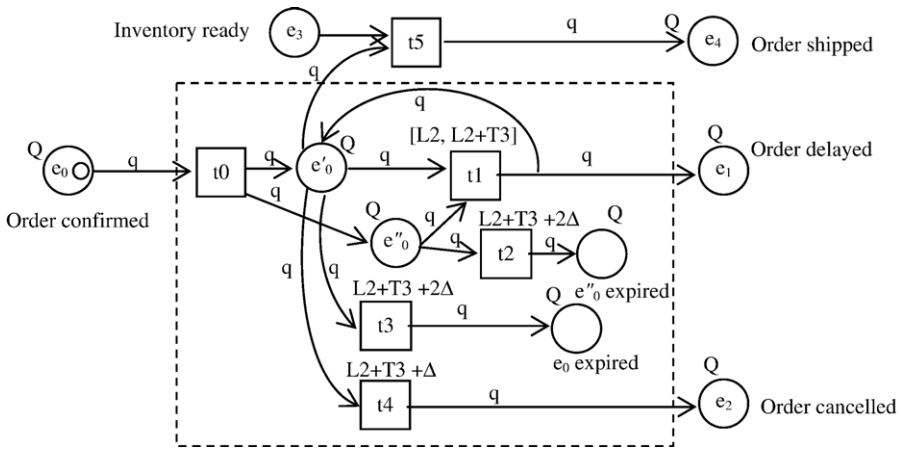


Fig. 5. Petri net model of an order process (Pattern 3).

time intervals could overlap; however, by ensuring the intervals are non-overlapping it would be possible to make a deterministic choice based on time. The following example illustrates this pattern.

Example 3: If an order, with lead time $L2$, has not been shipped (i.e., not consumed by some other rule) within time $L2$ after it is confirmed (e_0), the order is treated as delayed (e_1) (but e_0 is not consumed yet); however, if an order is delayed by more than time $T3$, it is treated as undeliverable and cancelled (e_2). (Perhaps the customer does not want it if the delay is more than $T3$. So e_0 is consumed at this time.) This example can be formulated as:

$$e_0(q, L2+T3+2\Delta)\{[\xrightarrow{[L2,L2+T3]} e_1(q)] \vee [\xrightarrow{L2+T3+\Delta} e_2(q)]\}$$

When an order is confirmed (see Fig. 5), a token is put in places e_0' and e_0''. Transitions $t1$, $t2$, $t3$, and $t4$ are enabled but do not fire at that moment. If this token is consumed by the shipment transition $t5$ before time $L2$ (relative to its arrival), transitions $t1$, $t3$, and $t4$ are disabled, but transition $t2$ will fire at time $L2+T3+2\Delta$ after the token arrival. Otherwise, if during the time interval $[L2, L2+T3]$ this token remains in place e_0', transition $t1$ will fire. After transition $t1$ fires, this token is immediately brought back to e_0' because some other rules (like $t4$) may use it later. If there is still a token in e_0' after $L2+T3$, transition $t4$ fires and produces event "order cancelled". Thus, the token in e_0' is consumed. In general, if this rule is triggered, it can produce two possible results: order delayed and cancelled or only order delayed, depending upon the temporal relationships. One can see this rule actually has complex semantics, yet its Petri net model can precisely describe such temporal relationships. Note that in Fig. 5, transition $t3$ never fires and it can be removed. We keep this transition in the figure for consistency with event semantics.

[P4] Pattern 4 (1 of N causes — single result Pattern): A result can have multiple (one or more) alternative causes. Hence, there is a need for this pattern, and its formal logical expression is as follows:

$$\{[e_1(n_1x_1, I_{10}) \xrightarrow{[I_{11},I_{12}]}] \vee [e_2(n_2x_2, I_{20}) \xrightarrow{[I_{21},I_{22}]}] \vee \ldots$$
$$\vee [e_m(n_mx_m, I_{m0}) \xrightarrow{[I_{m1},I_{m2}]}]\}e_0(x_0)$$

In this expression, the cause of event e_0 may be any one of e_1, e_2,…, e_m. Typically, this structure could be used to indicate the cause for an event. Fig. 6 is the Petri net representation of this structure, if every source event e_1, e_2,…, e_m is consumed by this rule. Note that the notion of expiration times can be applied to this pattern. For simplicity, the expiration times and expiration transitions are not shown in Fig. 6. For example, if transition $t1$ does not fire (because of insufficient tokens), e_1 will expire at time I_{10} by the firing of an expiration transition. The same standard simplification is applied to the next three patterns. A specific example of Pattern 4 is given as below.

Example 4: When a rush replenishment order is rejected (e_1), or delayed (e_2) by more than time $T4$ (if the

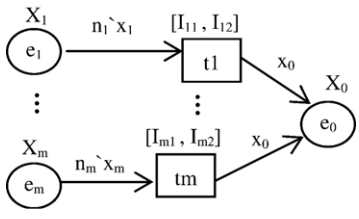


Fig. 6. Petri-net for *1 of N causes — single result* (Pattern 4).

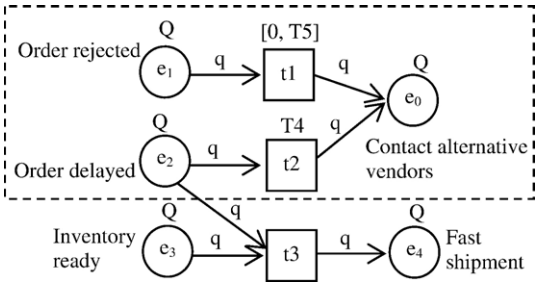


Fig. 7. Petri net of Example 4 (Pattern 4).

delay is less than $T4$, the delayed time can be compensated by faster shipment), contact alternative vendors (e_0). Logical form: $\{[e_1(q) \xrightarrow{T4}] \vee [e_2(q) \xrightarrow{[0,T5]}]\} e_0$.

As the Petri net model in Fig. 7 shows, in this example, if an order is rejected by a vendor an alternative vendor must be contacted in a short interval, say [0, $T5$]. If the order is delayed, a token is put into place e_2 immediately. How long this token remains in place e_2 is exactly the order delay time. If the order is delayed for time $T4$, then transition $t2$ has been continuously enabled for the same time, so transition $t2$ fires immediately. The fired transition means that, in order to replenish inventory in time, alternative sourcing is required. If the delay does not exceed time $T4$ and then the inventory is ready, a fast shipment (e_4) is used to compensate for this delay.

Therefore, through these examples we see that colored time Petri nets not only represent the transformation of events, but also simulate the underlying business activities. The latter advantage cannot be achieved by its logical formulations.

[P5] Pattern 5 (1 cause — N results Pattern): This pattern recognizes that a cause may have multiple consequences and captures all *concurrent* consequences of a particular event. Logically, this is expressed as: $e_0(n_0 x_0, I_{00}) \xrightarrow{[I_{01}, I_{02}]} \{e_1(x_1) \wedge e_2(x_2) \wedge \ldots \wedge e_m(x_m)\}$.

In this expression, n_0 occurrences of event e_0 generate m different events concurrently. Fig. 8 shows the Petri net representation of this rule. As Fig. 8 shows, the m events are represented as output places of transition $t1$, which is enabled by n_0 occurrences of input event e_0. Transition $t1$ fires within an interval [I_{01}, I_{02}] after n_0 tokens are placed

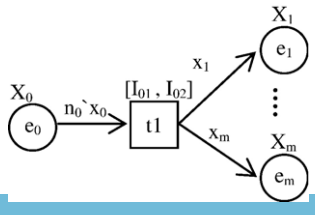


Fig. 8. Petri net for *1 cause — N results* (Pattern 5).

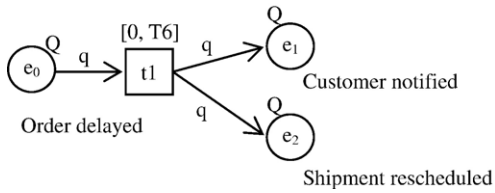


Fig. 9. Petri net of Example 5 (Pattern 5).

in e_0. In this case, this Petri net representation shows a *1 cause — N results* pattern.

Example 5: If an order is delayed (e_0), notify customer (e_1) *and* reschedule the shipment (e_2) immediately, i.e. $e_0(q) \xrightarrow{[0,T6]} [e_1(q) \wedge e_2(q)]$.

Fig. 9 is the Petri net model of Example 5. If $T6$ approaches 0, $t1$ fires instantaneously. This simple example illustrates that this structure can be used to present the concurrent events that originate from the same cause.

[P6] Pattern 6 (N causes — 1 result Pattern): This pattern is the reverse of the above pattern, and it is used to model the concurrent causes of a particular event. The following rule shows that there are m preconditions, e_1, e_2,…, e_m, which occur simultaneously to produce event e_0:

$$\{e_1(n_1 x_1, I_{10}) \wedge e_2(n_2 x_2, I_{20}) \wedge \ldots \wedge e_m(n_m x_m, I_{m0})\} \xrightarrow{[I_{01}, I_{02}]} e_0(x_0).$$

Assuming every source event is consumed by this rule, this formulation can be transformed into the Petri net of Fig. 10. This figure shows that each precondition can be modeled as an input place of transition $t0$, and the result e_0 is the output place of this transition. This Petri net exhibits an *N causes — 1 result* pattern as does the next example.

Example 6: When the shipper of a confirmed order (e_2) is not available (e_1), find another shipper (e_3) within a short time interval $T7$. This rule can be logically formulated as:

$$\{e_1(q) \wedge e_2(q)\} \xrightarrow{[0,T7]} e_3(q).$$

The Petri net of Fig. 11 shows the precise representation of this rule. In Fig. 11, two events e_1 and e_2 in conjunction produce one result, event e_3.

[P7] Pattern 7 (non-occurrence of an event pattern): The above patterns were all based on the occurrence of events. However, non-occurrence of an event can also signal valuable information and hence we need a pattern for that. Negation is usually used to express the non-occurrence of a particular event. Typically, non-occurrence of an event and occurrence of another event may, in conjunction, cause some other significant event to

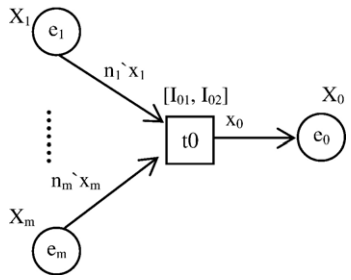


Fig. 10. *N causes* — 1 result Petri net (Pattern 6).

happen. The following logical formula describes this situation:

$$\{e_1(n_1x_1, I_{10}) \wedge [\neg e_2(n_2x_2, I_{20})]\} \xrightarrow{[I_{31}, I_{32}]} e_3(x_3)$$

Event e_1 and non-outcome of e_2 cause e_3. Actually, if event e_1 is consumed by this rule, this formulation can be transformed into a Petri net similar to Fig. 11, with the difference that the arc from place e_2 to transition $t1$ is replaced by an inhibitor arc as in Fig. 12 (see [6] also). In general, Fig. 12 and the Petri net representation of Example 7 below have a *non-occurrence* pattern.

Example 7: When an order arrives (e_1), if there is *no* out-of-stock (e_2) situation, the order is confirmed (e_3). Logically, this rule can be formulated as:

$$\{e_1(q) \wedge [\neg e_2(s, T2)]\} \xrightarrow{[0,T8]} e_3(q).$$

Fig. 13 is the Petri net model of Example 7. After the order arrives, a token is put into place e_1. At that time, if there is no token in place e_2 it means there is no out-of-stock event, and transition *t1* fires within a short time, say [0, *T*8], and puts a token in place e_3 to indicate the order is confirmed. Otherwise, if there is a token in place e_2, it inhibits the firing of transition $t1$.

In this section, we have developed 7 basic patterns that capture cause–effect relationships in Petri nets. Next, we will use an example to show that these patterns can be combined together as building blocks to create more complex Petri nets.

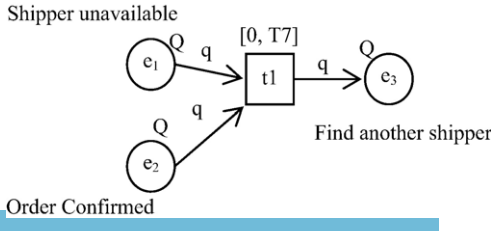


Fig. 11. Petri net of Example 6 (Pattern 6).

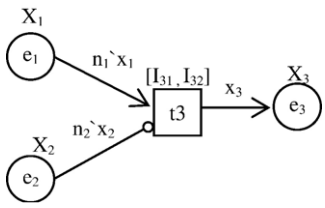


Fig. 12. Non-occurrence pattern (Pattern 7).

4.3. Composing new patterns and creating user-defined patterns

Above we discussed 7 basic patterns to capture complex cause effect relationships. Now we demonstrate how they can be combined to create new user-defined patterns. In general, patterns can be combined (or composed) if they have common input or output events (i.e., places that have the same label). By superimposing common places shared by existing patterns, new patterns can be created. This approach has been used in modeling logic programs as Petri nets [25]. Obviously, if two patterns do not share any events then they cannot be directly composed. The possible scenarios for pattern combination are as follows:

(1) The output place of one pattern is the input place of another pattern (sequential)
(2) The two patterns have one or more common input places (Parallel 1)
(3) The two patterns have one or more common output places (Parallel 2)
(4) The two patterns have common input *and* output places (Parallel 3)

When two patterns are composed in sequence, they form more complex cause–effect chains. On the other hand, if they share common inputs, the patterns will compete for firing by taking tokens from the common event places and will exhibit more complex interactions. A detailed analysis of the various possible interactions for the different combinations is beyond the scope of this paper; however, we will illustrate our approach by showing how two new, non-trivial and useful, user-defined patterns can be created.

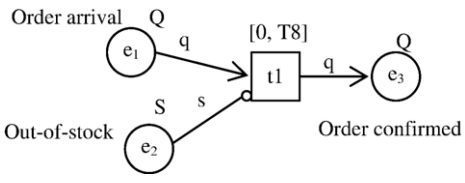
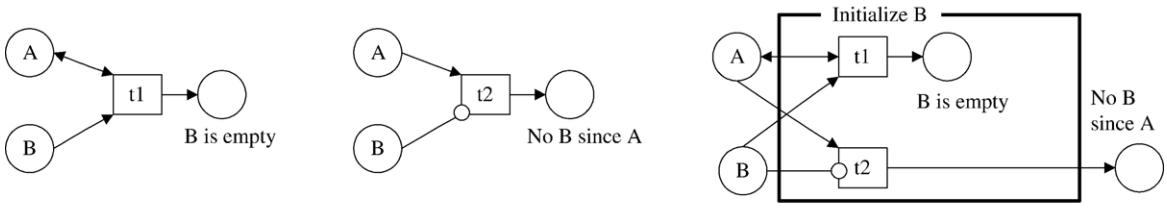


Fig. 13. Petri net of Example 7 (Pattern 7).



(a) Use Pattern 6 (b) Use Pattern 7 (c) Combine Patterns 6 and 7

Fig. 14. Composing a new *event initialization* pattern by combining Patterns 6 and 7.

First, we create a new pattern to "initialize *B* when *A* occurs", as shown in Fig. 14. Thus, when event *A* occurs, already existing occurrences of event *B* must be cleared. This *event initialization* pattern can be created by composing existing patterns as follows:

(1) Using Pattern 6, when event A happens, if any prior B events exist, they are cleared (Fig. 14(a)).
(2) Then, Pattern 7 is used such that transition *t2* fires when the place for event B is empty and puts a token in the place marked "No B since A" (Fig. 14(b)).
(3) Combine Pattern 6 and Pattern 7 by superimposing their common places for events A and B (Fig. 14(c)).

Similarly, in Fig. 15 we show how another new pattern called *consecutive events* can be created using this new *event initialization* pattern as a building block. The *consecutive events* pattern generates an exception event when events A and B (initialized after A) happen within a time interval *T*. To create this pattern, we first apply Pattern 6 to places "No B since A" and "B", as shown in Fig. 15(a). If tokens do not arrive in B within the required interval, say 2 time units, then tokens in place "No B since A" are said to expire. Later, we combine the new *event initialization* pattern with Pattern 6, as shown in Fig. 15(b). We can foresee various applications for these new patterns. For example, the consecutive events pattern could be used in a supply chain to notify the manager if a "machine breakdown" and "no shipment arrival" events occurred within 1 day. Similarly, in telecommunication applications also such consecutive events would be useful [7].

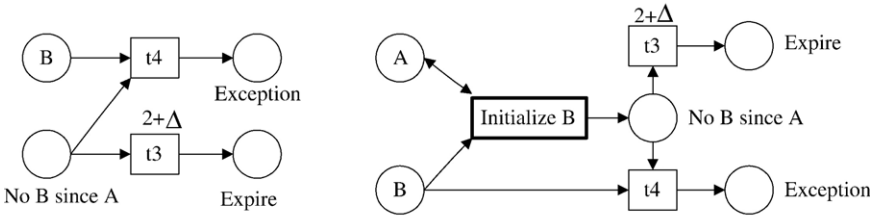
Clearly, although the seven basic patterns are not exhaustive, the ability to compose them and create new patterns is a powerful feature that allows us to model most realistic situations. Moreover, if necessary, new primitive patterns can also be created from scratch by giving their Petri net description.

5. An example of event causality analysis using Petri nets

In this section, we will first show a Petri net that is built using the above seven patterns in the context of a realistic supply chain scenario. Subsequently, we will analyze event causality by simulation and dependency graphs.

5.1. Scenario of events and rules for a complete Petri net

First, we will give an example scenario description. Suppose there is a Vendor Managed Inventory (VMI) arrangement between a distributor and a vendor. In this arrangement, the vendor manages the inventory level for the distributor, proposes the new supply orders



(a) Apply Pattern 6 (b) Combine *event initialization* and Pattern 6

Fig. 15. Composing a new *consecutive events* pattern from *event initialization* and Pattern 6.

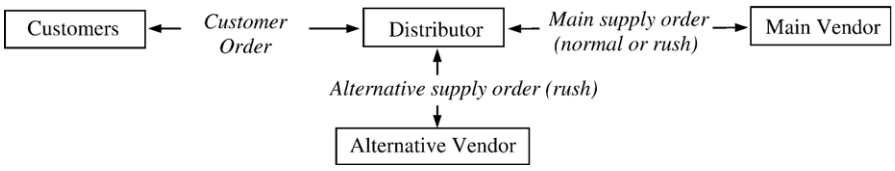


Fig. 16. Interactions between trading partners in the example supply chain.

to the distributor, and ships them after receiving the distributor's approval. The distributor sells products to its customers, and normally ships its customers' orders (in short, orders) from stock, but whenever there is an out-of-stock situation, a rush supply order is placed with the vendor. When there is more than one out-of-stock event in a week at the distributor, this situation should be considered as a supply chain exception and reported to the supply chain manager immediately. The vendor would usually respond to rush supply orders as soon as possible, but they may be rejected if there is a serious production delay. Moreover, in case of production delay, all supply orders may be delayed. The distributor can contact an alternative vendor for replenishment if its normal or rush supply order is delayed or rejected. Fig. 16 shows all the trading partners in such a supply chain. For simplicity, in this figure we assume there is one product, but one can similarly model multiple products also as we show in Section 6. First, we need to identify the events and then write the rules that connect them together. The events of interest are summarized in Fig. 17 and each event corresponds to a place in the Petri net.

Next, we consider the rules that relate these events to one another, and also refer to the corresponding patterns used for modeling these rules (in brackets).

1. When a customer order arrives (p1) and there is no out of stock (not p2), the order is confirmed (p6). [P7]
2. When a customer order arrives (p1) but there is an out-of-stock (p2), a back order (p3) is generated. [P6]
3. When a back order occurs, a rush supply order with lead time $L1$ is sent to the vendor (p4). [P1]
4. When the rush supply order is confirmed by the vendor (p5), the back order is also confirmed to the customer (p6). A back order must be confirmed within $L2+T3$, where $L2$ is the lead time of the back order, and $T3$ is the maximum allowed delay time; otherwise, it expires and is cancelled (p25). [P6]
5. If there is a production delay (p14), any incoming rush supply order is rejected (p13), because there is no production capacity left to fulfill any rush supply order in a short time. Otherwise, the rush supply order is confirmed. A production delay can be resolved in time interval $[a, b]$. [P6;P7]
6. A rush supply order is shipped during time $[0, L1]$ if there is no production delay (p14). [P7]
7. A production delay (p14) can cause a supply order delay (p15). If a rush supply order is delayed (p15) for more than time $T4$, it leads to unavailable inventory when customer order delivery is due (p17). [P5;P1]
8. If a rush supply order is rejected (p13) or delayed (p15) for more than time $T4$, contact alternative vendors for alternative sourcing (p16). [P4]
9. When a rush supply order is shipped (p18) by one of alternative vendors, the corresponding back

Place (or event) description

p1: Customer order arrival	p2: Out-of-Stock
p3: Back order	p4: Rush supply order
p5: Rush supply order confirmed	p6: Customer order confirmed
p7: Customer order delayed	p8: Notify customer of order delay
p9: Customer order cancelled	p10: Customer order shipped
p11: Out-of-Stock event expires	p12: Notify supply chain manager
p13: Rush supply order rejected	p14: Production delay
p15: Supply order delayed	p16: Contact alternative vendors
p17: Stock unavailable when delivery is due	p18: Rush supply order shipped
p19: Alternative sourcing failed	p20: Customer order rejected
p21: Production delay (p14) resolved	p25: Back order cancelled
p22: p2′ expired	p23: p3′ expired
p24: p5′ expired	
p26: p5″ expired	p27: p6′ expired
p28: p2″ expired	

Fig. 17. Possible events in the supply chain.

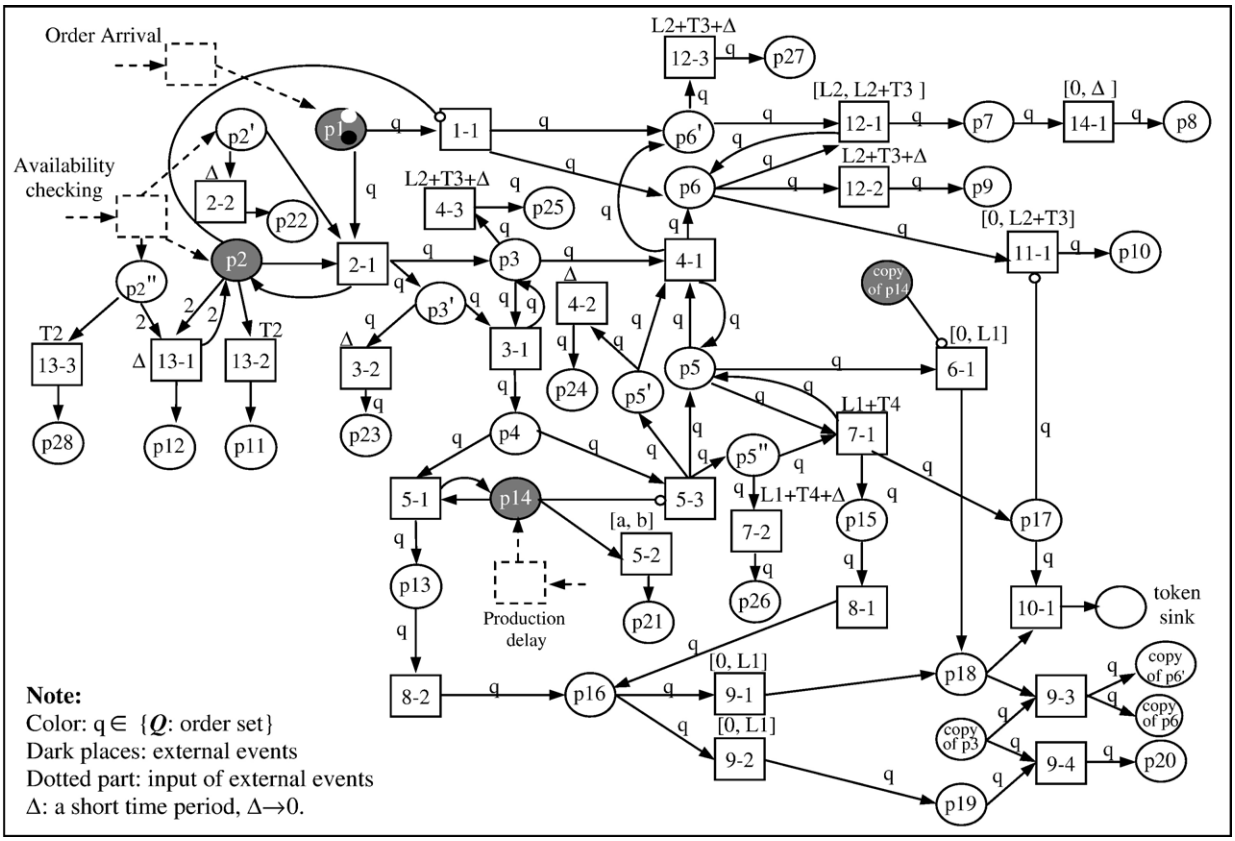


Fig. 18. A supply chain event Petri net.

order can be confirmed (p6) and shipped (p10); otherwise, the customer order can be rejected (p20). [P6;P1]

10. When a supply order is shipped from a vendor (p18), inventory is available for delivery (so if there is a token in p17, it is removed). [P6]
11. When delivery is due, if inventory is available (not p17), the order is shipped (p10). [P7]
12. a. If an order (with lead time *L2*) has not been shipped in time *L2* after it is confirmed (p6), the order is delayed (p7);
    b. If an order is delayed (p7) more than time *T3*, then the order is cancelled (p9). [P3]
13. If there are two unresolved out-of-stock events (p2) during time *T2*, the supply chain manager is contacted immediately (p12). [P2]
14. If the order is delayed (p7), notify the customer at time *T1* (p8). [P1]

The above 14 rules can be easily formulated in terms of colored time Petri nets as shown in Fig. 18. The dark places in the figure are the input events of this net. Place p1 contains two different tokens representing the two order arrivals. Events which are not consumed by event rules are transformed into multiple places, such as p2, p2′, and p2″, where p2 holds tokens for events, and the others are special mechanism for preventing repetitive firing of transitions.[3]

This Petri net was implemented using CPN Tools [23]. CPN Tools is a graphical computer tool supporting Colored Petri nets (CPN). However, since CPN Tools does not explicitly support time, a workaround is introduced to add temporal constraints to a transition.[4] Fig. 19(b) shows an implementation of Rule 1 and Rule 2 from the detailed supply chain example above (See Fig. 19(a)) in CPN Tools. In Fig. 19(b), "ORDER", "STOCKOUT", and "BACKORDER" are color sets of different places. Place p1 contains two tokens. For example, 1'(2, [*a*, *c*])@10 means there is one token (1') with color (2, [*a*, *c*]) and timestamp 10 (@10). Note that "2" is the order number (of integer data type) and [a,c] is a

[3] This point was explained in Section 4.1 in the non-consumption case.

[4] CPN Tools supports a notion of time. However, since it is an executable language allowing for automatic simulation it requires deterministic or stochastic time. Hence, the interval times are translated into guards based on an explicit clock.

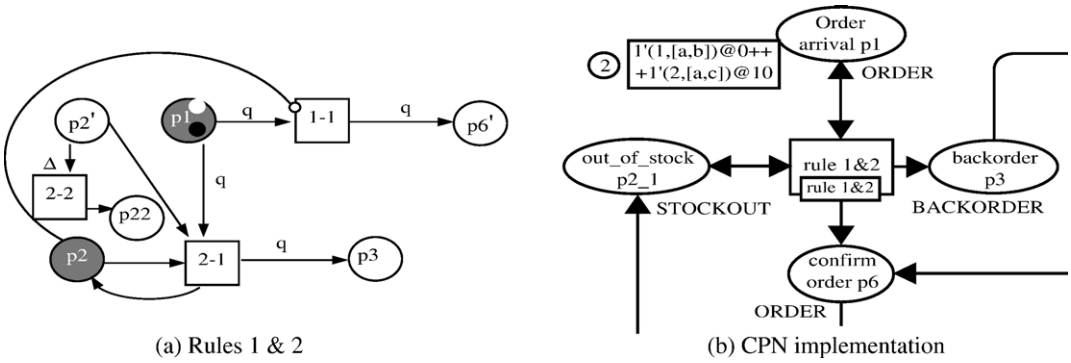


Fig. 19. Mapping rules 1 and 2 to CPN Tools (from the detailed example).

list of products (of list data type), which here denotes two products, "a" and "c" for order 2. In addition, Fig. 19(b) is a hierarchical Petri net. The label "rules 1 and 2" on the transition in the center of the figure refers to the sub-Petri net of Fig. 19(a). It should also be noted that an inhibitor arc is substituted by an equivalent structure with normal arcs, so inhibitor arcs also reflect causal relationships between events. For example, in Fig. 19(b), *p6* (confirmed order) depends on both p1 (order arrival event) and p2_1 (out-of-stock). More precisely, an occurrence of p1 and non-occurrence of p2_1 lead to p6.

Details of CPN Tools and hierarchical Petri nets can be found in [11]. Next, we will analyze events using *dependency graphs* based on running this model.

5.2. Dependency graph analysis

The Petri net shown in Fig. 18 can be considered as an "event machine," i.e. when fed with input events, it will generate a set of composite events (both intermediate and final), and show the causal relationships between them. The behavior of this "machine" for the life of a particular instance or for a given time period can be represented by a simple dependency graph [8]. A *dependency graph* is a cause–effect graph of events produced from one or more Petri net instances (such as an order) over a given time period. The dependency graph is created from the Petri net by using the rule that *the output event*(*s*) *of a transition depends upon its input event*(*s*).

By executing the Petri net model with actual case data, we can create dependency graphs to show causal relationships that actually transpired between events. Table 1 describes the sequence of event occurrences and the transitions that fire when the events take place. The relationships are reflected in Fig. 20 that shows an event dependency graph generated based on the Petri net of Fig. 18. The table also gives time values in the last column. These times are based on assigning suitable values for a hypothetical case to the parameters of Fig. 18 as follows (say, in days):

$$L1 = 20, L2 = 50, T1 = 1, T2 = 50, T3 = 20,$$
$$T4 = 10, a = 60, b = 80.$$

Fig. 20 enables us to analyze the various events and their causes. The events that represent *exceptions* are shaded in this figure. The consequences of a particular event can be traced forward along this directed graph, while its causes should be traced backwards until one or more root nodes are reached. For example, it is not difficult to see that $E8$ and $E12$ are the main causes of exception $E13$, i.e., product A was out of stock with the distributor and a rush supply order $R2$ was issued, but this rush supply order was rejected by the vendor because of a production delay. Similarly, the graph shows that the ultimate exceptions resulting from $E12$ are $E21$, $E22$ and $E24$. The sequence of main events is as follows:

Production is delayed ($E12$)→ rush supply order $R1$ is also delayed ($E15$)
→ another vendor is contacted ($E17$)
→ alternative sourcing failed ($E22$)
→ Order $O1$ cancelled ($E24$)

Moreover, notice that the exception $E11$ (notify supply chain manager) happens because of two stock-out events of product A within 50 time units as denoted by events $E2$ and $E8$. Thus:

Stock out of A for order $O1$ ($E2$) and Stock out of A for order $O2$ ($E8$)
→ Notify supply manager ($E11$)

Actually, Fig. 20 only shows one possible scenario and gives the ultimate disposition of orders $O1$ and $O2$: $O1$ was cancelled, while $O2$ was filled. Fig. 21 shows another out-of-stock situation during order

Table 1
A trace of possible event sequence generated from Fig. 18

Event	Description	Trans. fired	Place	Time
E1	Order *O*1 arrival	–	p1	0
E2	Out-of-Stock of product A (for *O*1)	–	p2	0
E3	*O*1 is on back order	1–1	p3	0
E4	Rush supply order *R*1 is placed for *O*1	3–1	p4	0
E5	Supply order *R*1 is confirmed to customer	5–3	p5	0
E6	Order *O*1 is confirmed	4–1	p6	0
E7	Order *O*2 is received	–	p1	10
E8	Product A is out-of-stock (for *O*2)	–	p2	10
E9	*O*2 is placed on back order	1–1	p3	10
E10	Rush supply order *R*2 is placed for *O*2	3–1	p4	10
E11	Contact supply chain manager	13–1	p12	10
E12	Product A production is delayed	–	p14	10
E13	Rush supply order *R*2 is rejected	5–1	p13	10
E14	Alternative vendor is contacted for *R*2	8–2	p16	10
E15	Rush supply order *R*1 is delayed for time *T*4	7–1	p15	30
E16	Product A is unavailable when *O*1 is due	7–1	p17	30
E17	Alternative vendor is contacted for *R*1	8–1	p16	30
E18	Rush supply order *R*2 is shipped from the alternative vendor (i.e., non-occurrence of event "product unavailable when *O*2 due")	9–1	p18	30
E19	Order *O*2 is confirmed	9–3	p6	30
E20	Order *O*2 is shipped	11–1	p10	31
E21	Order *O*1 is delayed	12–1	p7	50
E22	Alternative sourcing attempt for *R*1 failed	9–2	p19	50
E23	Notify customer about order *O*1 delay	14–1	p8	50
E24	Order *O*1 is cancelled	12–1	p9	71

fulfillment; however, now the outcome is different. Here, *E*16 ("product A is unavailable when *O*1 is due") is resolved by *E*18 ("rush supply order for *R*2 shipped from the alternative vendor"). Therefore, order *O*1 is shipped (*E*20) within its lead-time. Later on, in spite of *E*21 ("alternative sourcing for *R*1 fails"), rush supply order *R*1 is shipped (*E*22) from the main vendor after some delay. Eventually, order *O*2 is also fulfilled (*E*24) by the incoming inventory from rush order *R*1. The modified events for this scenario are shown in Table 2 (events *E*1 through *E*17 are the same as in Table 1). Fig. 21 shows the new dependency graph for these events. Nevertheless, *E*11 ("notify supply chain manager") still occurs as before.

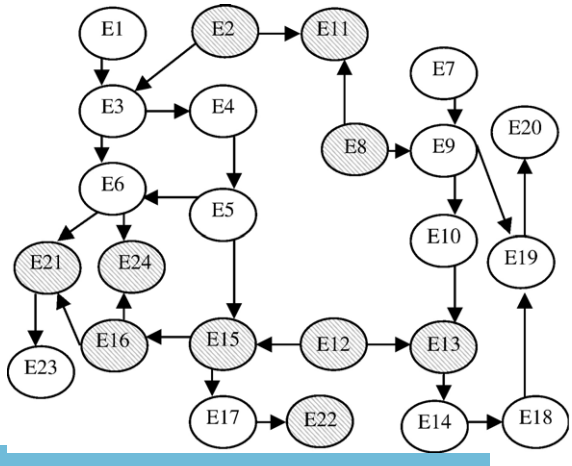


Fig. 20. Dependency graph of Table 1 (exceptions are shaded).

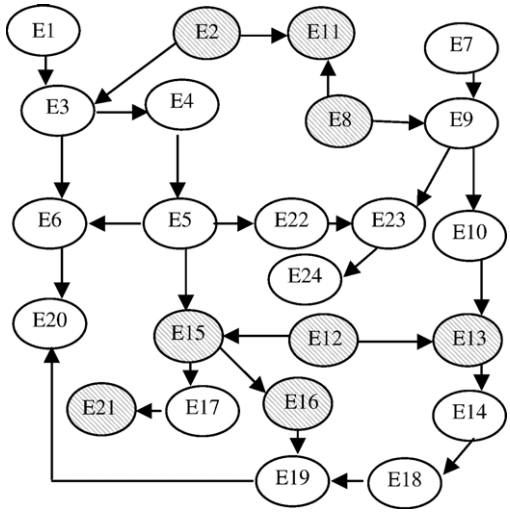


Fig. 21. Dependency graph of Table 2.

Table 2
An alternative scenario of events generated from Fig. 18

Event	Description	Trans.fired	Place	Time
... Events *E*1 thru *E*17 are same as in Table 1 ...				
E18	Rush supply order for *R*2 shipped from the alternative vendor.	9–1	p18	30
E19	Product available for *O*1 (token in p17 removed) (i.e., non-occurrence of event "product unavailable when *O*1 due")	10–1	p17	30
E20	Order *O*1 shipped	11–1	p10	30
E21	Alternative sourcing for *R*1 fails	9–2	p19	50
E22	*R*1 Shipped from the main vendor	6–1	p18	80
E23	Order *O*2 confirmed	9–3	p6	80
E24	Order *O*2 shipped	11–1	p10	80

These two dependency graphs show only two of many possible scenarios and serve to illustrate our approach. The advantage of this approach is that using a Petri net model as an event machine we can generate dependency graphs to predict and analyze different "interesting" scenarios. Moreover, by playing "token games", supply chain managers can explore a large number of possible event dependency graphs which lead to desirable results (e.g. order filled successfully) or significant exceptions (e.g., order cancelled). The design of an algorithm or heuristic that can automatically generate dependency graphs containing such events of interest is left as a future exercise. In this context it should be noted that while it is possible to analyze all possible dependency graphs using reachability analysis techniques [3] for time colored Petri nets, it is not very feasible to do so for large problems for complexity reasons, and hence heuristic techniques are required. Next, we provide a summary of simulation results and analyze their implications for supply chain management.

6. Simulation results

To demonstrate the practical value of our approach, a detailed simulation experiment was conducted. In this simulation, we generated a large number of customer order arrival events and traced the order fulfillment process in terms of times of occurrence of each event. To make the simulation realistic, we assume there are three products, say A, B, and C. In general, more products can also be supported. Table 3 shows the parameters of our simulation experiment.

Table 3
Simulation parameter settings

Parameter name	Value or distribution
Set of items in a customer order	Random selection from three products: A, B and C
Customer order inter-arrival time	Exponential distribution with mean of 7 time units
Prob. of successful alternative sourcing (PSAS)	0.5
Inter-arrival time between production delay events	Exponential distribution with mean of 100
Resolution time for production delay (RT)	Uniform distribution range [60, 80]
Normal supply arrival schedule	2 arrivals for each item every 30 time units

The simulation runs for a period from 0 to 3500 time units. 500 customer orders are generated and processed. Among them, 445 orders were successfully shipped, and the other 55 orders were cancelled or rejected because of out-of-stock events and failures to find alternative sourcing. In Table 4, the "baseline case" column summarizes the number of main events generated during the simulation interval. Table 4 also shows that although about one quarter of customer orders (135 out of 500) occur in stock out situations, yet most of them (84 out of 135) can still be successfully fulfilled through rush supply orders. In addition, about 10% of customer orders (48 out of 500) are fulfilled by alternative sourcing, which shows that alternative sourcing is important.

Table 5 shows the detailed distribution of out-of-stock events by product. Each product accounts for about one third of these 182 out-of-stock events. In practice, it may be difficult for a supply chain manager to trace each one of these 182 events individually. Using rule 13 (see Section 5.1) we can filter these events and reduce the number of events sent to the manager. Thus, the supply chain manager may be notified only when there are *two out-of-stock events* within a 50 time unit interval. Therefore, the number of events which need management attention is reduced to 80, about 40% of the original number of events. Moreover, the manager can adjust Rule 13 to further reduce this number suitably.

In addition, the events in Table 4 can be used to calculate key performance indexes of the supply chain. As Table 4 shows, the fill rate of customer orders is 89% and the average time between an order arrival and the shipment of the order is 28 time units. In addition, on average, it takes 54 time units for the main vendor to replenish rush

supply orders, because production delays occur frequently (35 delay events) and they last a while before being resolved. In contrast, it takes a shorter average time (10 time units) to get supplies from alternative vendors. In general, since the customer order fill rate is somewhat low, the performance of this system may need to be improved. We next show how this can be done with our approach.

An important aspect of our approach is the ability to do sensitivity analysis. To show how such analysis can help to improve the performance of this supply chain, we alternately considered the effect on performance of changing two parameters: reducing the resolution time of production delays (Strategy 1), and increasing the probability of finding alternative sourcing (Strategy 2).

Table 4
Comparing different strategies in terms of events

Events	Baseline case	Strategy 1	Strategy 2
	RT=[60, 80], PSAS=0.5	RT=[30, 50]	PSAS=0.7
Order arrivals (p1)[a]	500	500	500
Customer order shipped (p10)	445	473	475
Customer order cancelled (p9)	4	3	1
Customer order rejected (p20)	47	21	20
Back order cancelled (p25)	4	3	4
Out-of-stock events (p2)[a]	182	182	182
Production delay (p14)[a]	35	35	35
Customer order delayed (p7)	4	4	1
Back order (p3)	135	135	135
rush supply order (p4)	135	135	135
rush supply order fulfilled	84	111	111
by main vendor	36	77	33
by alternative vendors	48	34	78
Rush supply order rejected by main vendor (p13)	102	60	102
Supply order delayed (p15)	8	16	6
Contact alternative vendors (p16)	110	76	108
Alternative sourcing failed (p19)	62	42	30
Performance Indexes			
Customer order fill rate	89%	95%	95%
Average customer order fulfillment time	28	27	28
Average replenishment time of rush supply orders (main vendor)	54	18	27
Average replenishment time of supply orders (alternative vendors)	10	10	11

[a] These are input events. The three strategies have the same input events.

Table 5
Numbers of out-of-stock events

Products	A	B	C	Total
Out-of-stock events	53	66	63	182
Notify supply chain manager of out-of-stock events	24	29	27	80

Strategy 1 considers the possibility that a production delay can be resolved in a time interval [30, 50] instead of [60, 80]. For Strategy 2, another alternative vendor is introduced into the supply chain so that the probability of finding alternative sourcing is increased to 0.7. The simulation results of these two strategies are also shown in Table 4. Using Strategy 1, although there is a large number of back orders, more than a half of them (77 out of 135) are still delivered through successful rush supply orders from the main vendor while only 34 back orders are replenished by alternative vendors. For the second strategy, 70% of back orders (78 out of 111) are fulfilled by alternative vendors. Both strategies lead to an increase in the fill rate of customer orders. Thus, compared with the baseline strategy, Strategy 1 and Strategy 2 can increase the fill rate to 95%. Similarly, other scenarios can be explored and analyzed in detail with this technique.

7. Comparison with related work

Related research for detailed modeling of supply chains is still limited. Active databases rely on event-condition-action (ECA) rules [18]. Such rules make databases "active" by allowing them to react to events, i.e., when an event occurs, if some conditions hold, an action (such as database update, insert, query) is taken. In [10], ECA rules are used to specify exceptions and exception handling approaches in the context of workflow management so that recent cases that are similar can be solved with reference to these rules. However, the drawback of ECA rules is that they cannot do event chaining in a natural way, and hence cannot easily facilitate the analysis of cause–effect relationships between events. Moreover, they are also unable to trace back the causes of events, or forecast future events. Finally, temporal attributes cannot be modeled explicitly.

One domain in which event management has been studied with considerable interest and success is the area of network management. Here the objective is to manage a large number of low-level events that may be related and to extract high-level events that require management attention while ignoring the unimportant ones. Hasan et al. [9] provide a conceptual framework for describing causal and temporal relationships between network

events. In [8], Gruschke give a dependency graph based algorithm for event correlation in networks. This algorithm is used to map raw events in the network to faulty objects based on the links in the graph. These approaches are relevant in supply chains also, but they lack a precise representation of temporal constraints.

In [5], Time Petri nets are integrated into databases and used for semantic mapping of events in computer networks. The transitions are associated with guard conditions expressed as database constraints. It is an interesting approach with possible applications in supply chains, but harder to implement and verify. In particular, there is no standard approach to transform an event rule to a Petri net and, moreover, time constraints are captured in an ad-hoc way. Case-based approaches for event correlation in networks are given in [14]. These methods compare a new case against a database of cases and look for stored solutions; however, they require an application-specific model and are computationally complex. Rule-based or knowledge-based approaches are discussed in [7,30]. Here the knowledge of the expert is described in rules and the rules are applied to diagnose a new problem. However, formal representations of rules are not provided, and hence it is difficult to extend these approaches to other domains.

Other related work includes a proactive SCEM system with agent technology discussed in [4]. While it focuses on event monitoring and alert generation, this system lacks the capability of analyzing events and suggesting solutions. Patterns have been studied in many domains, but the ones developed in the context of workflow management [28,24,27] are the closest to our work. However, they do not address the complex temporal constraints. Classical Petri nets have been used to model rules in knowledge bases [15,20,31]. However, high-level time colored Petri nets are naturally more expressive because, besides capturing temporal constraints elegantly, they can support a rich vocabulary of event rules, such as sequence operators (*and*, *or*), modifiers (*last*, *nth*, *any*, *none*), and predicates [7]. We have modeled the part of this vocabulary that is relevant in supply chains, such as *and*, *or*, *any*, and *none*, in our seven basic patterns because our focus is on the most common patterns that arise in supply chains. The other part of the vocabulary consisting of modifiers can also be modeled as a further extension using the concepts of guards or multi-set colors in Petri nets (see [11,12]).

8. Conclusions

We developed an approach for modeling event relationships in a supply chain through Petri nets. The formalism consists of seven basic patterns that capture cause-effect relationships in Petri nets. These patterns can be combined together as building blocks to create other patterns and also more complex Petri nets. We used a very extensive example to illustrate this approach and showed in detail how dependency graph analysis can help to determine causal relationships between events in a dynamic supply chain. It should be noted that these relationships are complex and depend upon the exact timing of events. We demonstrated that slight changes in temporal relationships can result in a very different dependency graph and also final outcome.

Petri net simulation offers a mature technique for analyzing the Petri net models, and the easy availability of many Petri net software packages is an asset. We implemented Petri net models using CPN tools and performed sensitivity analysis by simulation. By changing a specific event parameter, such as event resolution time, we can show how supply chain performance is affected. Such scenario analysis can suggest solutions to improve supply chain performance. Therefore, by managing events, we can actually manage supply chain performance. We ran comprehensive simulation experiments illustrated how this approach can help decision makers to improve supply chain performance.

In summary, as supply chains get more tightly integrated across partners, it is becoming increasingly important to respond in real-time to events through a real-time sense-and-respond capability. We described a novel approach to model event relationships in a supply chain using Petri-net patterns that can be combined to create realistic Petri-net models of supply chains. We further implemented a model in a CPN tools, and ran simulation experiments with it. A unique feature of the approach is that the Petri nets are constructed from patterns or building blocks which can be composed together and extended to create new user-defined patterns. In future work, we would like to develop more formal verification techniques for the supply chain models, and also develop heuristics for reachability analysis of dependency graphs to predict "interesting" events.

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